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196. Proposed by L. E. NEWCOMB, Los Gatos, California.

Find the  $r$ th term of  $\left(x - \frac{1}{x}\right)^n \equiv z^n$  in terms of  $z$ .

I. Solution by G. W. GREENWOOD, B. A. (Oxon), Professor of Mathematics and Astronomy, McKendree College, Lebanon Ill.

Let  $x = \cos\theta + i\sin\theta$ , then  $z = 2i\sin\theta$ , and  $x^p = \cos p\theta + i\sin p\theta$ . This enables us to express any term of  $(x - 1/x)^n$  in terms of  $\theta$ , i. e., in terms of  $z$ .

II. Solution by F. D. POSEY, A. B., San Mateo, Cal.; G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and GRACE M. BAREIS, Bala, Pa.

$$x - 1/x = z, \therefore x = \frac{z \pm \sqrt{z^2 + 4}}{2}.$$

The  $r$ th term of  $(x - 1/x)^n$  is  $\frac{(-1)^{r-1}n(n-1)\dots(n-r+2)}{(r-1)!} x^{n-r+1} \left(\frac{1}{x}\right)^{r-1}$

$$= \frac{(-1)^{r-1}n(n-1)\dots(n-r+2)}{(r-1)!} x^{n-2r+2}$$

$$= \frac{(-1)^{r-1}n(n-1)\dots(n-r+2)}{(r-1)!} \left(\frac{z \pm \sqrt{z^2 + 4}}{2}\right)^{n-2r+2}.$$

197. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

$$\text{Solve } (18)^{4(2-x)} = (54\sqrt{2})^{3x-2}.$$

Solution by W. W. LANDIS, Dickinson College, Carlisle, Pa.

Writing the equation in the form  $(1)\dots 18^{4(2-x)} = 18^{3(3x-2)}$  we find  $x = \frac{2}{17}$ .†

Also solved by G. W. Greenwood, J. E. Sanders, A. H. Holmes, F. D. Posey, R. A. Wells, G. I. Hopkins, H. R. Higley, G. B. M. Zerr, E. L. Sherwood, Grace M. Bareis, J. Scheffer, L. E. Newcomb.

## GEOMETRY.

219A. Proposed by H. F. MacNEISH, A. B., Assistant in Mathematics, University High School, Chicago, Ill.

Draw through a given point a line which shall divide a given quadrilateral into two equivalent parts: (1) when the point lies in a side of the quadrilateral; (2) when the point is without; (3) within the quadrilateral.

\*Dr. Zerr also gives the values

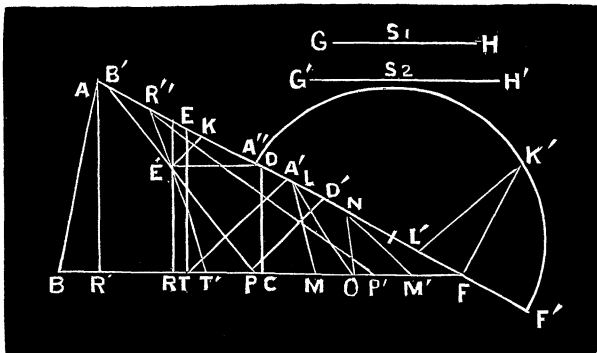
$$(-1)^{r-1} {}_n C_{r-1} (z + 1/z - 1/z^3 + 1/z^5 - \dots)^{n-2r+2},$$

$$(-1)^r {}_n C_{r-1} (1/z - 1/z^3 + 1/z^5 - \dots)^{n-2r+2}.$$

†Owing to the periodicity of the exponential function  $a^y$  may be written  $e^{y \log a + 2n\pi i}$  ( $n$  = integer). It now follows from (1) that  $e^{\frac{1}{2}(22-17x) \log 18 + 2n\pi i} = 1$ , whence  $\frac{1}{2}(22-17x) \log 18 + 2n\pi i = 0$ , and therefore  $x = \frac{2}{17} + (4n\pi i / 17 \log 18)$ . When  $n=0$ ,  $x = \frac{2}{17}$ . ED.

## II. Solution by L. E. NEWCOMB, Los Gatos, California.

(1) Let  $ABCD$  be the quadrilateral of area  $S^2$ ,  $E$  the given point in the side  $AD$ ; complete the triangle  $ABF$  and let  $S_1^2 = \frac{1}{2}S^2 + \text{area } DCF$ ;  $S_2^2 = \text{area } ABF = S^2 + DCF$ ;  $GH = S_1$ ,  $G'H' = S_2$ . Draw  $ER$  and  $AR'$  perpendicular to  $BF$ , of lengths  $d$  and  $d'$ , respectively. On  $FA$  lay off  $FL = S_2$ ,  $LN = S_1$ ,  $FB' = FB$ ,  $FA' = d'$ ,  $FD' = d$ ; on  $FB$  lay off  $FM = S_1$ . Join  $LM$  and parallel to  $LM$  draw  $NO$ ; join  $LO$  and parallel to  $LO$  draw  $B'P$ ; join  $PD$  and parallel to  $PD'$  draw  $A'T$ ; join  $ET$ . The line  $ET$  divides  $ABCD$  equally.



follows easily,  $S_2^2 : \frac{1}{2}d' \cdot BF = S_1^2 : \frac{1}{2}d' \cdot FP$ . But  $B'F (=BF) \cdot \frac{1}{2}d' = S_2^2$ , hence  $\frac{1}{2}d' \cdot FP = S_1^2$  and  $FD' (=d) : FA' (=d') = FP : FT$  and thus  $\frac{1}{2}d' \cdot FT = \frac{1}{2}d' \cdot FP$ . Since  $\frac{1}{2}d' \cdot FP = S_1^2$  it follows that  $\frac{1}{2}d' \cdot FT = S_1^2 = \frac{1}{2}S^2 + \text{area } DCF$ , and consequently  $\frac{1}{2}d' \cdot FT - \text{area } DCF = \text{area } ETCD = \frac{1}{2}S^2$ .

(2) Let  $E'$  be the given points without the quadrilateral. For the construction of a line through  $E'$  cutting off a given area  $S_1^2$  from the triangle  $ABF$ , see Geometry, Problem No. 218.

(3) Let  $E''$  be the given point within the quadrilateral; draw  $E''A''$  parallel to  $BF$ ,  $E''K$  perpendicular to  $AF$  and let  $E''K = a$ . Extend  $AF$ , the distance  $2a$ , to  $F'$ . On the diameter  $A''F'$  describe the semi-circle  $A''K'F'$ ; draw  $FK'$  at right angles to  $AF'$ ; from the center  $K'$  with radius  $S_1$  describe an arc cutting  $AF'$  at  $L'$  and join  $L'K'$ . On  $FB$  lay off  $FM' = a$ ,  $FP' = S_1 - L'F$ ; join  $NM'$ , and parallel to  $NM'$ , draw  $P'R''$ . Through  $R''$  and  $E''$  draw  $R''T'$  and let the altitude of the triangle  $R''T'F$  to the base  $R''F = b$ .  $R''T'$  is the required line.

In proof,  $E''K (=a) : b = R''A'' : R''F$ , hence  $b \cdot R''A'' = a \cdot R''F$ ; also  $M'F (=a) : NF (=S_1) = FP' (=S_1 - L'F) : R''F$ , hence  $a \cdot R''F = S_1 \cdot FP' = S_1 \cdot (S_1 - L'F)$ . Now  $L'F = \sqrt{[S_1^2 - (FK')^2]}$  and  $(FK')^2 = 2a \cdot A''E = 2a \cdot (R''F - R''A'')$ , therefore

$$a \cdot R''E = S_1(S_1 - \sqrt{[S_1^2 - 2a(R''F - R''A'')]}), \text{ consequently } \left( \frac{a \cdot R''F}{S_1} - S_1 \right)^2 = S_1^2 - 2a \cdot R''F + 2a \cdot R''A'', \text{ hence } \frac{a^2 \cdot (R''F)^2}{S_1^2} = 2a \cdot R''A'', \text{ and } a \cdot R''F = \frac{2R''A'' \cdot S_1^2}{R''F}. \text{ But}$$

$a \cdot R''E = b \cdot R''F$ , hence  $b \cdot R''A'' = \frac{2R''A'' \cdot S_1^2}{R''F}$  and  $b \cdot R''F = 2S_1^2$ .  $\therefore \frac{1}{2}b \cdot R''F = S_1^2 = \frac{1}{2}S^2 + \text{area } DCF$ . Consequently  $\frac{1}{2}b \cdot R''F - \text{area } DCF = \frac{1}{2}S^2 = \frac{1}{2}$  of area of the quadrilateral  $ABCD$ .